Sliding Behavior of Rubber on Snow and Concrete Surfaces

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Introduction

The sliding movement of a highly deformable body like a rubber block on any surfaces can be viewed as a rather complicated mechanism. Depending on the characteristics of the sliding surface and the environmental temperature the influence of temperature is accounted for on the basis of the existence of master curves for the frictional behavior [1].

On non-deformable, smooth surfaces, e.g., wet concrete surfaces, the load transfer capacity between the rubber block and the surface is determined by a frictional process. The frictional effect can be split into an adhesive and a hysteretic part depending on the smoothness of the surface [2]. Further dependencies are the contact pressure between the rubber block and the sliding surface, the sliding velocity of the rubber block and the environmental temperature. The influence of temperature is accounted for on the basis of the existence of master curves for the frictional behavior [3]. On deformable surfaces, e.g., snow surfaces, a further effect can be observed. Due to the geometry changes (deformation of the sliding surface) an additional component of the traction force is activated.

In the following section a realistic material model for snow-like materials is described. The parameters of the model were calibrated by means of experiments. The model was used in a numerical investigation concerning the sliding behavior of a rubber block on a snow surface. The results were compared with experimental results. To get a better insight of the frictional behavior on “hard” surfaces, e.g., wet concrete surfaces, a special testing device was used and a comprehensive experimental investigation was carried out. Based on the obtained results a numerical model for the frictional behavior was developed.

Sliding behavior on snow surfaces

During the sliding process on snow surfaces large deformations of the sliding surface itself can occur. Therefore, it is important to describe the material behavior of snow realistically. A viscoplastic “critical state” material model is used to model the constitutive behavior of snow. This model for soil-like materials is formulated on the basis of multi-surface plasticity theory within the framework of finite deformation plasticity at large strain. It is characterized by two hardening/softening mechanisms and a set of yield functions proposed in the following forms [1]

\[ \Phi_\varepsilon(l_1, l_2, q_s(x_s)) = \sqrt{\frac{c_\varepsilon^2}{q_\varepsilon} (l_1 + q_s)^2 - \kappa_c (T - l_1)} - \sqrt{c_t q^3} \leq 0, \quad (1) \]

\[ \Phi_\varepsilon(l_1, q_s(x_s)) = l_1/3 - q_1 \leq 0. \quad (2) \]

\[ l_1, l_2 \] are the first invariant of the Kirchhoff stress tensor and the second invariant of the Kirchhoff stress deviator. \( q_s(x_s) \) is the hardening (softening) parameter dependent on the strain-like internal variable \( x_s(x_s) \) associated with the yield surface \( \Phi_\varepsilon(\Phi_\varepsilon) \). \( c_\varepsilon \) determines the shape of \( \Phi_\varepsilon \). \( \kappa_c \) and \( T \) denote material parameters related to the angle of friction and the cohesion of the material.

\( \Phi_\varepsilon \) denotes a so-called “tension-cut-off” plane perpendicular to the hydro-
static axis. In case of triaxial states of tensile stresses, this plane moves towards the origin, associated with a volumetric softening process. U constitutes a smooth “drop-shaped” yield function closed along the compressive and the tensile “side” of the hydrostatic axis (Figure 1). Its shape in the stress space changes continuously in the course of hardening.

The calibration of the parameters of the material model for snow-like materials was done by numerical re-analysis of experiments, such as hydrostatic compression tests and shear box tests for different snow materials.

For the performed traction tests of rubber on a pre-compressed snow surface with a density of $\rho \sim 0.5 \text{ g/cm}^3$ a rubber block was first subjected to a constant normal pressure. Then the rubber block was moved over the surface with a constant velocity of $v = 14.4 \text{ mm/min}$. Before the movement a consolidation phase limited to 10 minutes was provided. During the consolidation and the movement phase relatively large deformations of the snow were observed.

For the numerical investigation the multi-purpose FE code MARC was used [4]. The mechanical behavior of the rubber tread was described using a Mooney-Rivlin material model. The classical Coulomb friction law is used to approximate the behavior of the interface between rubber and snow [1].

Figure 2 shows the deformed FE mesh as well as the distribution of the vertical normal stress at a movement of the block of $u = 1.728 \text{ cm}$. In this example a soft rubber compound was examined. In Figure 3 various load-displacement curves from the FE analyses (FEA) are given for the same soft rubber compound. The difference of the thick solid line and the dashed line can be viewed as the influence of the so called “edge effect”. As a numerical experiment a friction coefficient of $\mu = 0.0$ was used in the contact area between the rubber block and the deformable snow surface. The thin solid line shows the result. A very regular stick-slip motion was observed. Beginning with a displacement of $u = 3 \text{ cm}$ the form of the FE mesh and therefore the numerical response changed.

Sliding behavior on concrete surfaces

Experimental Investigation

In the past tire tests were widely used to obtain information about the friction behavior of rubber on different surfaces. In such tests the frictional material properties are always influenced by non-reproducible test conditions as well as by several structural effects of the tire and its tread patterns. The elimination of these deficiencies was the motivation for the development of the Linear Friction Tester by Continental AG, Germany, and by the Institute for Strength of Materials at Vienna University of Technology [5]. With this testing device reliable friction coefficients, $\mu$, of rubber specimens under various conditions at different road surfaces can be measured.

Linear Friction Tester (LFT)

The recently adapted Linear Friction Tester (LFT) [5] is used for investigations of
the basic mechanisms of viscoelastic rubber friction. Here, a combination of the adhesive and hysteretic part of the friction behavior is considered. Depending on the smoothness of the friction surface the adhesive or hysteretic part of friction prevails. The friction behavior is basically influenced by the type of friction surface, tread geometry, pressure, sliding velocity and temperature. The LFT is a very flexible testing equipment which allows variation of the experimental parameters within a wide range.

In Figure 4 a drawing of the front view of the LFT is shown. It consists of a linear drive which is attached to a stiff steel frame. A servo amplifier with a high positioning accuracy controls the linear drive. At the movable leading sledge of the linear drive the load application system including the mounting for the test specimen (see Figure 5) and the measuring systems for vertical and horizontal forces and displacements are mounted. A plane friction surface can be fixed at a bracket which is adjustable in height.

As indicated in Figure 6 the experimental procedure consists of the following steps:

- acceleration of the test specimen to a prescribed sliding velocity,
- touch-down and vertical loading of the specimen,
- movement of the loaded rubber specimen on the friction surface with a constant sliding velocity up to a distance of 200 mm, and
- lift-up and retardation of the specimen.

The vertical movement and loading of the specimen is done by means of a pneumatic cylinder. The resulting vertical displacements are measured with an LVDT position transducer. The sliding velocity as well as the horizontal movement of the leading sledge are controlled by pulse counting of the linear drive. The friction coefficient

\[ \mu = \frac{F_H}{F_V} \]  

observed during a friction test is recorded and displayed continuously. In Eq. (3) \( F_H \) denotes the horizontal load component in the contact interface between the test specimen and the friction surface. \( F_V \) is the vertically applied load. The forces are measured by means of three independent load cells.

The LFT enables measurement of the sliding load transfer of rubber treads on plane hard surfaces, such as wet and dry asphalt and concrete, as well as on soft surfaces like compressed snow and ice. The nominal vertical pressure on the rubber blocks with a contact area of up to 40 cm² can be varied from 0.5 to 8 bar. The sliding velocity can be prescribed without any restriction between 0 and 1000 mm/s. The LFT is located in a climate chamber. For this reason friction tests can be carried out at en-
Environmental temperatures ranging from \(-25^\circ C\) to \(+35^\circ C\).

**Experimental Results**

A comprehensive experimental investigation was carried out in order to obtain a reliable data base for the friction properties of rubber under various conditions. Friction experiments on ice and wet concrete were performed with different tread geometries and rubber compounds for a nominal pressure of \(p = 1, 2, 3, 4,\) and \(5\) bar, for sliding velocities of \(v = 1, 10, 100,\) and \(1000\) mm/s, and for environmental temperatures of \(T = -15, -5, +5, +15,\) and \(+25^\circ C\).

The dependence of friction coefficient \(\mu\) on \(p\) and \(v\) is shown in Figure 7 as an example for tests on a wet concrete surface at an environmental temperature \(T = 18^\circ C\). In this 3D-graph a maximum friction coefficient \(\mu = 1.04\) can be observed for a sliding test specimen under \(p = 1\) bar and \(v = 10\) mm/s.

The results obtained from experiments form a basis for the numerical investigation. The experimental data were used for calibration of the parameter of a numerical friction law by means of a least-square fitting procedure. As an alternative a genetic algorithm was used very successfully for the identification of the parameters of the friction law [6].

\[
\tau = (\alpha |p|^\beta + |\beta|p) + |\tau| \leq 0. \quad (4)
\]

In Eq. (4) \(\tau\) denotes the traction shear stress, \(\alpha, \beta\) and \(n\) are parameters of the interface law, which depend on the surface properties. The dependence on the environmental temperature is taken into account by means of the WLF theory [10]. Additionally, the different frictional behavior in the case of a braking situation is considered.

The implementation of the friction law was done analogous to a plasticity material model by means of the return mapping algorithm. Therefore, the tangential sliding velocity is split into an elastic part and an inelastic part, \(\dot{g}_N = \dot{g}_N^{el} + \dot{g}_N^{in}.\) With this kinematic split, the slip condition, e.g. Eq. (4), and the non-associated slip rule for the inelastic part of the tangential velocity,

\[
\dot{g}_N^{in} = 0; \quad \dot{g}_T^{in} = \lambda \dot{\tau}_T^{in} = \lambda \text{ sign } (\tau) \quad (5)
\]

with the consistency parameter \(\lambda,\) the return mapping algorithm can be summarized as: (1) evaluation of the elastic predictor, (2) back projection to slip surface if slip condition is violated, and (3) determination of the consistent linearization.

The frictional behavior changes completely during an acceleration process and a following deceleration process. This was verified by means of only three experiments on the LFT. In Figure 8 the results are shown in form of a \(\mu - v\) diagram. With increasing sliding velocity the friction coefficient is increasing until about \(v \approx 300\) mm/s, then the friction coefficient is decreasing to a relatively constant value. This constant value is then kept in the case of decreasing sliding velocity. This behavior is accounted for in the developed friction law.

**Numerical Investigation**

For the numerical investigation two commercial, multi-purpose FE programs were used (ABAQUS [7], MARC [4]). With both FE codes an Ogden material model was chosen for the rubber compounds [8]. As a conclusion of the experimental investigation a friction law dependent on normal pressure, sliding velocity and environmental temperature was developed. It is based on the pressure dependent modified Coulomb friction law proposed by [9].
The deformed test specimen and the stresses normal to the contact surface in a stationary sliding state obtained by MARC are shown in Figure 9. The according pressure distributions are illustrated in Figure 10 obtained by ABAQUS. For these examples the following parameters were used: sliding velocity \( v = 300 \text{ mm/s} \), a constant vertical nominal pressure \( p = 4 \text{ bar} \) and an environmental temperature of \(+18 \, ^\circ C\).

Summary

A constitutive model for snow-type materials was developed. It can be described as a viscoplastic “critical state” material model which is characterized by two hardening/softening mechanisms and a set of yield functions, i.e., multi-surface yield functions, formulated within the algorithmic framework of finite deformation elasto-viscoplasticity. Based on experimental data from uniaxial compression tests, hydrostatic tests, and creep tests of snow, the calibration of the material parameters was performed. This constitutive model was used to describe the deformation behavior of the sliding surface. During the sliding process of a rubber block over a snow surface large deformations were observed. These deformations are responsible for an additional resistance against the sliding motion, which can be viewed as an “edge effect”. This effect was evaluated numerically.

The frictional behavior on “hard” surfaces was taken into account numerically by means of a friction law depending on the sliding surface, normal pressure, sliding velocity, and the environmental temperature. Therefore, a comprehensive experimental investigation with the so-called Linear Friction Tester (LFT) was performed. This testing device was especially developed to determine the frictional behavior of rubber blocks on various surfaces under different conditions of loading, temperature, and sliding velocity. The experimental results were directly used to determine the parameters of the proposed friction law.

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