Optimisation of nonlinear Material Parameter in uniaxial Compression Tests of Elastomer Specimen involving Friction

Elastomer · rubber · material parameter · reverse engineering · nonlinear optimisation · compression test

The determination of mechanical material parameters of elastomers is not easily possible. This is due to the fact that mechanical properties strongly depend on the composition of the underlying rubber compound and its manufacturing process. Furthermore, in uniaxial compression tests, data free of friction is required to gain realistic hyperelastic material parameters. The procedure presented here is characterised by the deployment of suitable optimisation tools until the simulation results coincide with comparable low friction test results. This method proves to be much more time and cost efficient rather than elaborate low friction compression test setups eventually yielding similar results.

Optimierung von nichtlinearen Materialparametern in uniaxialen Kompressionstests von Elastomerproben unter Einbeziehung von Reibung

Elastomer · Gummi · Materialparameter · Reverse-Engineering · nichtlineare Optimierung · Drucktest

Die Ermittlung der mechanischen Materialparameter von Elastomeren ist nicht ohne weiteres möglich. Dies liegt einerseits daran, dass die Eigenschaften der zu Grunde liegenden Mischungen stark von ihrer Zusammensetzung und dem Herstellprozess abhängen. Andererseits werden für die realitätsnahe Anwendung hyperelastischer Materialmodelle u. a. reibungsarme einachsige Drucktests benötigt. Das hier vorgestellte Verfahren basiert auf einer computerbasierten Optimierung zunächst reibungsbehafteter Materialparameter. Durch die Optimierung werden die Parameter so lange variiert, bis die Simulation zu vergleichbaren Ergebnissen führt wie ein reibungsarmer Testaufbau. Diese Methode ist sehr viel zeit- und kosteneffizienter als der rein experimentelle Ansatz, der am Ende vergleichbare Ergebnisse liefert.

Introduction

Elastomers consist of various compound ingredients, which usually do not underlie any norm but the specifications of the particular manufacturer, instead. Furthermore, the properties of the end product highly depend on the manufacturing process. The corresponding mechanical material parameters are therefore often unknown as well as not automatically available. Nevertheless, it is important to gain knowledge of these parameters in order to accurately describe the mechanical behaviour of critical industrial components made from elastomers. In particular the material parameters can be derived by means of stress-strain-curves that depend on various experimental test configurations. The minimum requirements to generate such curves are uniaxial tensile and compression tests which can represent homogeneous stress states. For a more detailed account on test requirements and the theory of nonlinear elasticity in general, the interested reader is referred to [1].

Preferably, the test samples come from finished components, due to the fact that the vulcanisation process and therefore the resulting material behaviour is also influenced by the part geometry. For tensile tests it is easily possible to measure homogeneous stress states. In that case the test samples just need to be long enough in order to respresent a homogenous stress state at the center of a test specimen. A subsequent determination of the associated material parameters does not represent any problems.

In contrast, the friction in between the clamping plates and the test specimen in compression tests leads to intensly inhomogeneous stress states. However, the adaptation of the mechanical material parameters assumes fully homogeneous stress-strain-curves which ideally require friction free test setups in compression. In general it is very challenging if not even impossible to provide compression test setups that can replicate homogeneous stress states. Friction based interferences are always present to a certain extent and distort the resulting material parameters.

The computer based optimisation methods presented in this paper allow the retrieval of realistic mechanical material parameters even for friction based uniaxial compression tests. It proves to be very time and cost efficient and forms an economic way for realistic simulations of critical elastomer components.

Basic remarks

The stress-strain curve obtained from an uniaxial compression test (Fig. 1) provide the starting point of our investigations. By means of a suitable curve fitting procedure, such a measured curve can be transformed into nonlinear material parameters that are deployed for subsequent finite element analyses. The underlying material models can accurately describe the resulting finite elastic strains and are based on strain energy potentials. To name just a few, models named after Yeoh, Arruda-Boyce and Neo Hooke belong to that class of so called hyperelastic strain energy functions.

Uniaxial compression test

In a specific uniaxial compression test configuration the test specimen made of elastomer material is compressed up to 50% of its initial shape (Fig. 3). This re-

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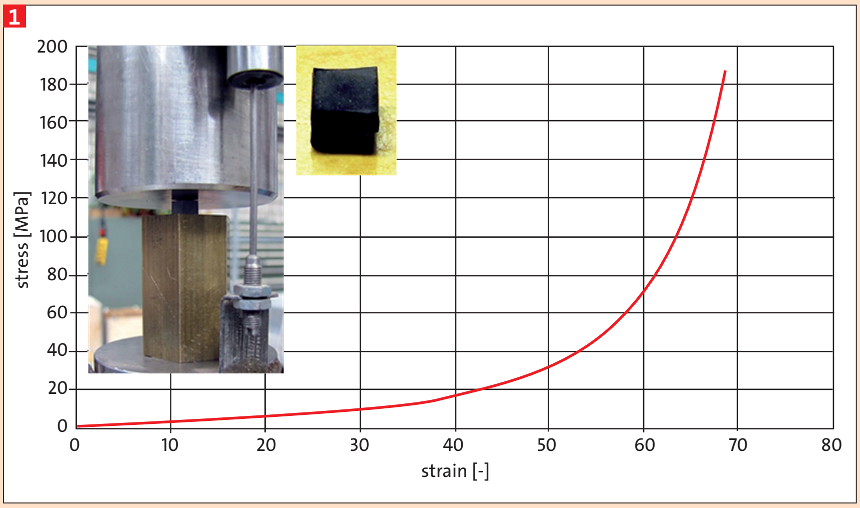

Fig. 1: Typical stress-strain-diagram of a rubber test.

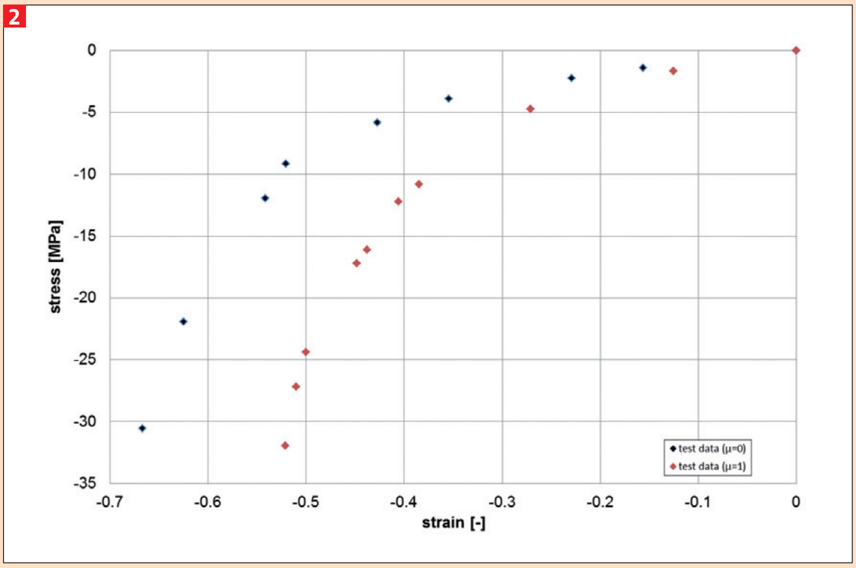

Fig. 2: stress-strain-curve of test data; black μ=0, red μ=1.

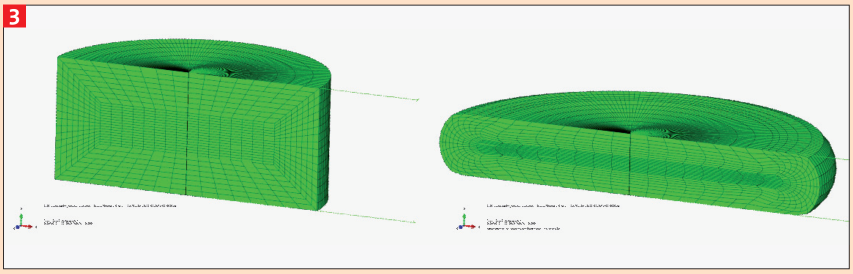

Fig. 3: test setup, compression of puck of 50 % from 12.7 mm to 6.35 mm.

sults in friction based stress-strain curves as depicted in Fig. 2.

The friction coefficients of μ = 0 and μ = 1, respectively, show two typical results from measurements. In addition, the reaction force acting on the test specimen during deformation is recorded for μ = 1, which will be used later on in the optimisation process (Fig. 6). As mentioned above a friction coefficient of μ = 0 can hardly be realised in measurements and therefore the deformed specimen exhibits a strongly inhomogeneous stress state as indicated by the deformed configuration of the test specimen in (Fig. 3).

Yeoh material model

The Yeoh model will be applied for material parameter optimisation throughout this paper. It is based on a strain energy potential expressed by means of a 3rd order polynomial [2].

$$U = \sum_{i=1}^{3} C_{i0} (\bar{I}_1 - 3)^i + \sum_{i=1}^{3} \frac{1}{D_i} (J_{el} - 1)^{2i}$$

This model is bases on phenomenological observations and fits for large strains as well as a small amount of test data (uniaxial tensile and compression tests). The coefficients C_{i0} and D_i represent material constants. J_{el} represents the elastic volume strain, whereas the total change in volume J and the first strain invariant $\bar{I}_1$ are calculated as follows [3]:

Total change in volume J

$$deformation\ gradient: \boldsymbol{F} = \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{X}}$$

$$\mathrm{J} = \det(\mathbf{F})$$

Note: J_{el} = J, if thermal expansion of the rubber material is neglected.

First strain invariant $\bar{I}_1$

$$\bar{\boldsymbol{F}} = J^{1/3} \boldsymbol{F}$$

$$left\ Cauchy - Green - tensor: \bar{\boldsymbol{B}} = \bar{\boldsymbol{F}} \bar{\boldsymbol{F}}^T$$

$$\bar{I}_1 = trace \bar{\boldsymbol{B}}$$

Curve fitting

The measured stress-strain data is fitted by the previously mentioned Yeoh material model using Abaqus [3] . Hence, the coefficients C_{10}, C_{20} and C_{30} are obtained. For the coefficients D_1, D_2 and D_3 representing volumetric deformations assumptions are made. The result of the curve fitting is shown below (Fig. 4):

C_{10} = 1.92444
C_{20} = 0.368634
C_{30} = .152256
D_1 is approximated by a value of 0.001, representing quasi incompressible behaviour of rubber materials, D_2 and D_3 are set equal to zero.

In the majority of cases these values follow a specific pattern according to Abaqus Theory Guide. Regarding those observations, C_{10} is usually the largest value coefficient of the three. In other words, if one of the coefficients is known, the order of magnitude of the other two shall be embedded in a range as listed below:

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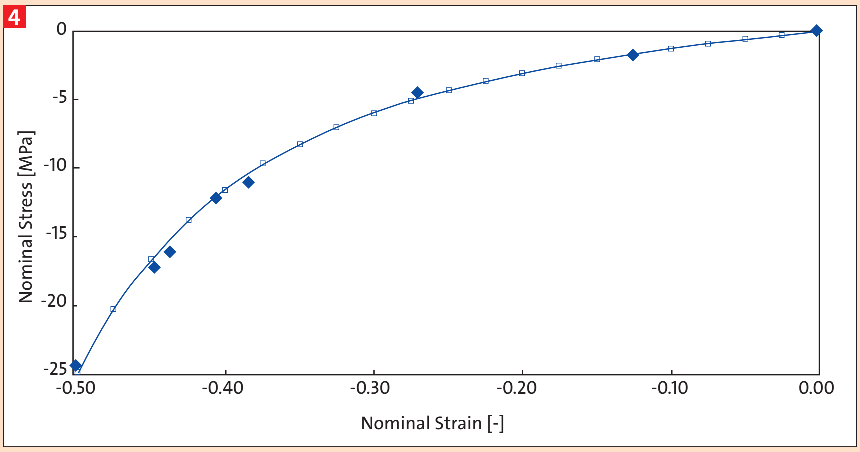

Fig. 4: curve fitting (blue line) in Abaqus/CAE according to the Yeoh model using the test data (black diamonds).

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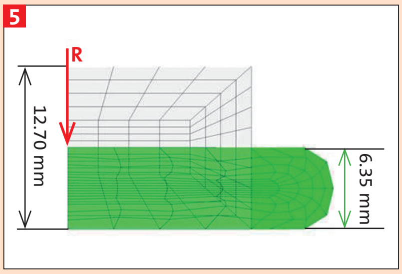

Fig. 5: FE analysis in Abaqus, imaged initial situation (grey) and deformed puck (green).

If $C_{10} = O(1)$

Then (typically) $C_{20} = -O(E\text{-}1 - E\text{-}2)$ and

$C_{30} = O(E\text{-}2 - E\text{-}4)$

Furthermore, as already mentioned in the introduction, a part of this inadequacy results from friction occurring in the test. Since the curve fitting of all material models is based on homogenous stress states, test data not being free of friction violates this assumption.

The values found by the curve fitting process above do not reflect this common pattern. Regarding these criteria, only the order of magnitude of C_{20} in relation to C_{10}. However, C_{20} is not negative and C_{30} is not located in the predefined magnitude range, either. Because of that it is reasonable to assume that the values C_{i0} do not describe the real behaviour of the scrutinized material. A confirmation to that is found in the FE analysis below.

Finite element analysis

The FE analysis represents the geometry of the uniaxial test setup. In Fig. 5, the result of this simulation is shown for a friction coefficient μ = 1. The output file also contains the reaction force R, indicated by the red arrow.

The curve fitting of the parameters for the Yeoh material model – this also holds for other hyperelastic material models – begins with the assumption that the boundary conditions are free of friction (homogeneous stress state). If this restriction is fulfilled in the test, no problems occur in the calculation of the parameters. Otherwise, the FE simulation will result in a wrong outcome. Even if the simulation operates with the same friction that occurred during the test, the result will be distorted if parameters were obtained from test data involving friction (see Fig. 6 for the difference between the reaction force of the material test and the one by FE analysis).

Optimisation process

Overview

In order to revise this inadequacy between the material test and the FE analysis, the flowchart in Fig. 7 shows the procedure of the optimisation. Initially, a curve fitting procedure is started for retrieving material parameters, meaning compression test data with friction is used for this purpose (step 1). These parameters serve as start values for the subsequent optimisation process. In a first FE analysis, the reaction force over time will be computed (step 2). Subsequently, this reaction force will be compared to the one that was recorded during the material test (step 3). The difference (Fig. 9) between the two enables the optimisation software [3] to vary the parameters within a limited range in such a way that a pre-defined objective function will be minimized (step 4). Afterwards, the abort criterion will be checked, which is given by the user. If the curves approach each other sufficiently accurate, the optimised pa-

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time	force
0.00000E+000	-0.00000E+000
1.56250E-003	-4.41934E+000
3.12500E-003	-9.82346E+000
5.46875E-003	-1.85270E+001
8.98437E-003	-3.20025E+001
1.42578E-002	-5.25483E+001
2.21680E-002	-8.37442E+001
3.40332E-002	-1.31182E+002
5.18311E-002	-2.03685E+002
7.85278E-002	-3.15460E+002
1.18573E-001	-4.90259E+002
1.78641E-001	-7.69784E+002
2.68742E-001	-1.23292E+003
3.68742E-001	-1.82101E+003
4.68742E-001	-2.51672E+003
5.68742E-001	-3.38767E+003
5.93742E-001	-3.69218E+003
6.03117E-001	-3.82066E+003
6.17180E-001	-4.01973E+003
6.38274E-001	-4.33415E+003
6.69914E-001	-4.84541E+003
7.17375E-001	-5.71604E+003
7.88567E-001	-7.31889E+003
8.88567E-001	-1.03960E+004
9.13567E-001	-1.13653E+004
9.22942E-001	-1.17547E+004
9.26457E-001	-1.19036E+004
9.31731E-001	-1.21572E+004
9.33708E-001	-1.22636E+004
9.36674E-001	-1.24247E+004
9.41124E-001	-1.26690E+004
9.47798E-001	-1.30436E+004
9.57809E-001	-1.36244E+004
9.72826E-001	-1.45412E+004
9.95352E-001	-1.60295E+004
1.00000E+000	-1.63550E+004

Name: time_force_data

	X	Y
1	0	-0
2	0.025	-154.931
3	0.0296875	-185.504
4	0.034375	-216.263
5	0.0414063	-262.762
6	0.0519531	-333.326
7	0.0677734	-441.062
8	0.0915039	-607.157
9	0.1271	-867.543
10	0.180493	-1286.73
11	0.260583	-1995.94
12	0.360583	-3072.59
13	0.460584	-4499.02
14	0.560583	-6582.96
15	0.585584	-7270.37
16	0.623083	-8617.92
17	0.637146	-9228.78
18	0.65824	-10234.9
19	0.68988	-11985.3
20	0.737341	-15293.8
21	0.808533	-22449.5
22	0.908533	-39917.6
23	1	-69626.3

Quantity Types

X: Time Y: Force

Fig. 6: record of the reaction force of the compression test (left hand side) and of the FE simulation (right hand side).

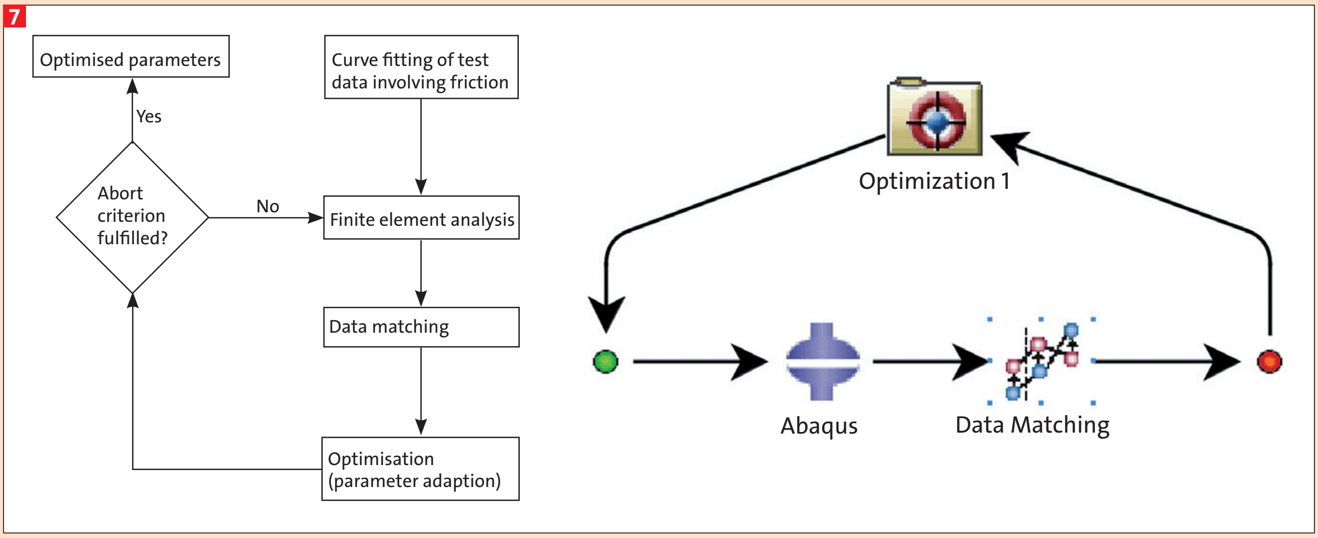

Fig. 7: flowchart of the optimisation process (left hand side) and the setup in the optimisation tool (right hand side).

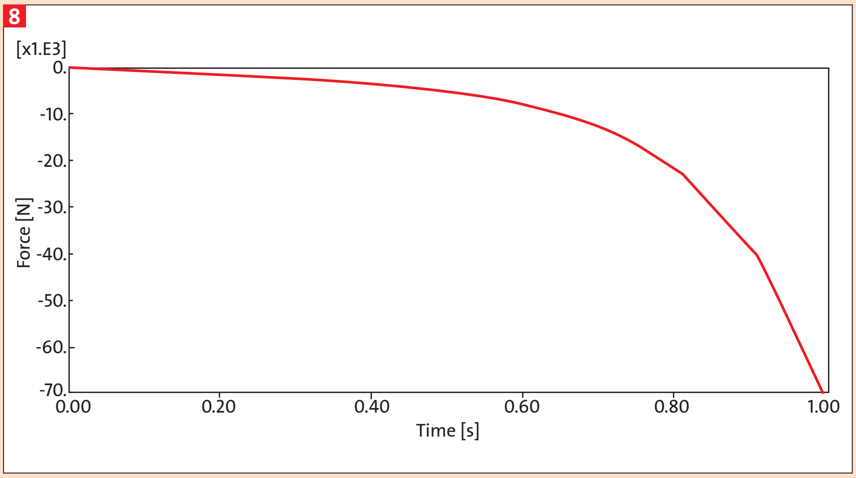

Fig. 8: Reaction force over time of test specimen as result of the FE analysis.

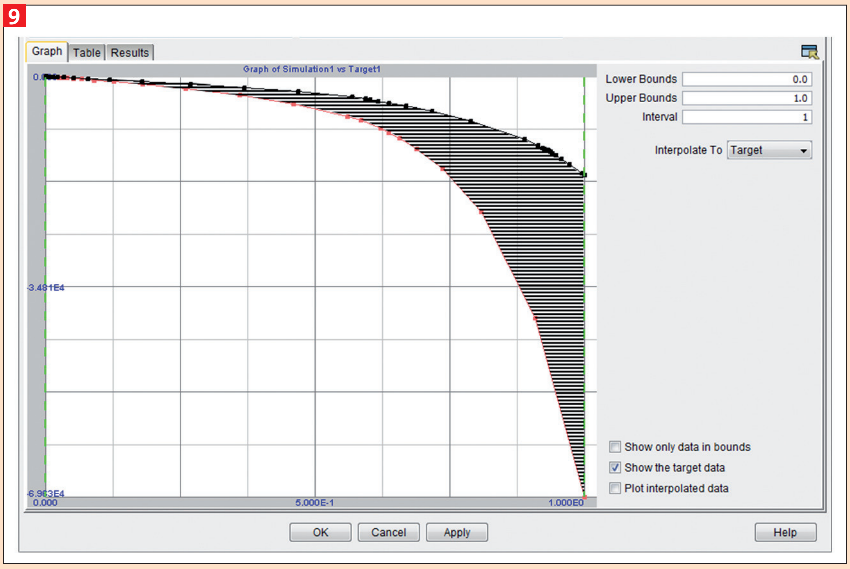

Fig. 9: difference (hatched area) between of the reaction force of the compression test (black) and the FE simulation (red).

rameters will be displayed. If not, steps 2 to 5 will be repeated until the abort criterion is fulfilled. Once the optimisation procedure has converged, the optimised material parameters lead to similar results in a FE analysis as those obtained from a zero friction compression test and therefore correspond to the reality. In short form the optimisation procedure reads as follows:

Step 1
Curve fitting of material parameters from compression test data including friction.

Step 2
FE simulation of the compression test setup including analysis of reaction force

Step 3
Comparison of reaction forces obtained from previous FE analysis and those from compression test. Due to curve fitting of stress-strain-curves involving friction, the FE simulation is too stiff initially (see Fig. 9).

Step 4
Parameter variation with pre-defined constraints for minimisation of an objective function (e.g. the hatched area in Fig. 9). These variations will be repeated as often as the abort criterion of the objective function will be fulfilled (see Fig. **10**).

The parameters are optimised according to the Hooke-Jeeves method. This algorithm contains iteratively made explorative and heuristic pattern moves. At first, the vicinity of the starting point is explored systematically to find the best point around the current point ($F(x)=f(x\pm\Delta)$). Thereafter, the heuristic pattern move is performed. In case that the modified, new point comes up with a better value, this point is going to be the new starting point for another explorative move. If the modified value doesn't lead to a better result, Δ is multiplied by α until a better point (value) is found. This procedure goes on as long as $\Delta > \varepsilon$. Epsilon is given by the user and functions as an abort criterion, because the algorithm itself doesn't observe the optimum [4, 5].

Function f to be varied is given by:

$$f(C_{10}, C_{20}, C_{30}, t) = A(reaction\ force\ FEA)$$

Objective function to be minimized:

$$A(reaction\ force\ FEA) - A(reaction\ force\ compression\ test\ data) \rightarrow 0$$

with $\Delta = 0.02$, $\varepsilon = 1 \cdot 10^{-6}$, $\alpha = 0.5$

Results

The final plot of the optimisation process shows that the curved of the two reaction forces eventually coincide (see Fig. 11).

The following parameters belong to the optimised graph in blue:

$C_{10} = 1.1696$
$C_{20} = 0.0936$
$C_{30} = 0.0$

Concerning the behaviour pattern mentioned in paragraph curve fitting, the optimised parameters better fulfil the order of magnitude criteria. The magnitude of C_{20} is two orders of magnitude smaller compared with C_{10} but it shows a positive algebraic value rather than a negative one. The third parameter, C_{30}, changed from the same order of magnitude as C_{20} to a smaller one. In particular this parameter decreases to a value between 0 and 6.25E-4, which is a good fit to the criterion of C_{30} stating it two to four orders of magnitude smaller than parameter C_{10}.

The objective function in the optimisation process was the minimisation of the area difference between two curves. The difference value was reduced from 8510.35 to 34.38. The whole process was

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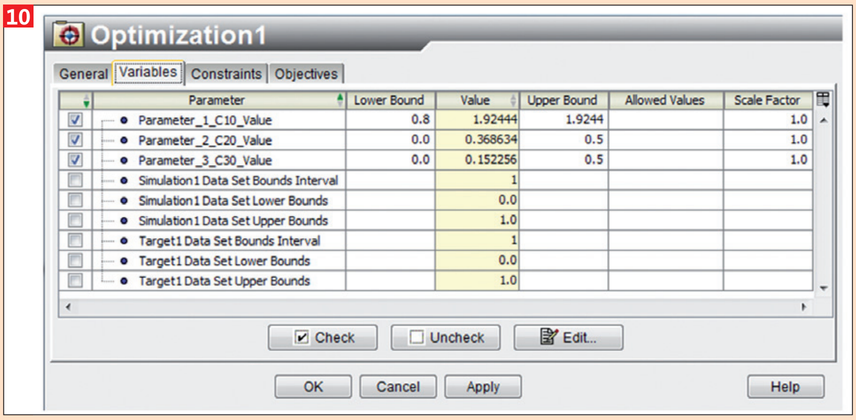

Fig. 10: parameter and constraint definition in the optimisation tool.

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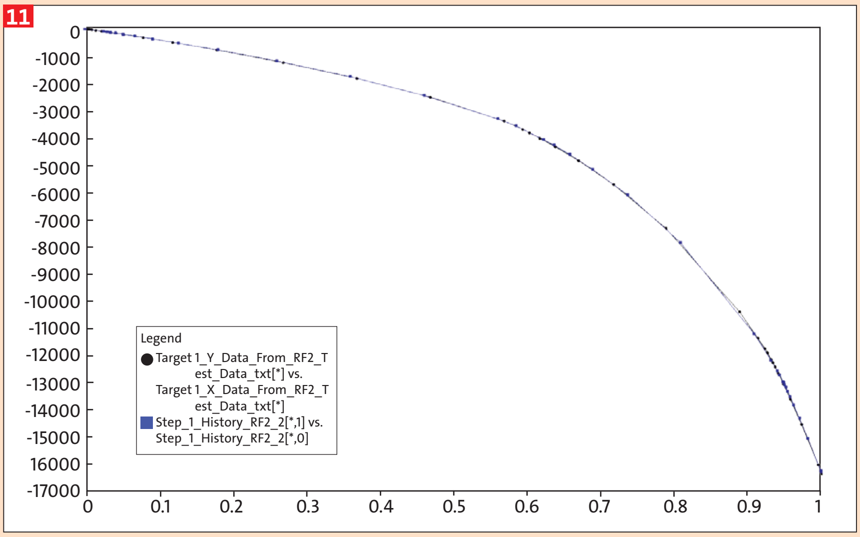

Fig. 11: superposition of the reaction force of the compression test (black) and of the FE analysis with the optimised material parameters (blue).

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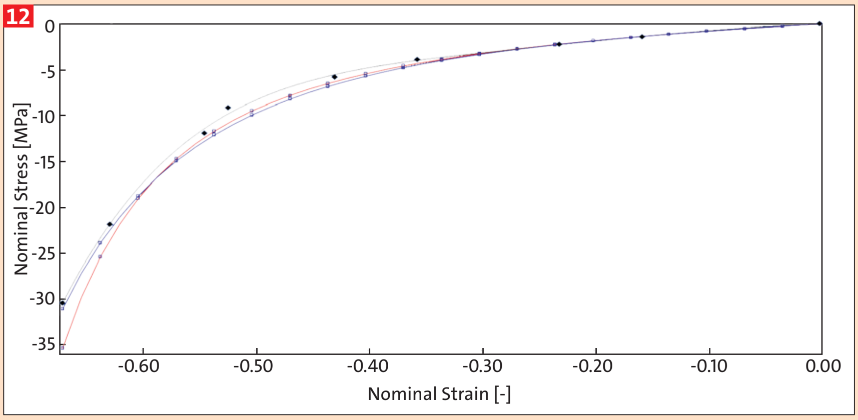

Fig. 12: Comparison of nominal stress / nominal strain; test data (black), Aabqus/Isight (blue), AnsysWB/OptiSlang (red).

1 List of starting and optimised material parameters

	C1	C2	C3	D1¹	D2¹	D3¹
Parameters based on test data	1.9244	0.3686	.1523	0.001	0	0
Optimised parameters using Abaqus and Isight	1.1696	0.0936	0.0	0.001	0	0
Optimised parameters using AnsysWB and OptiSlang	1.1767	0.0225	0.0228	0.001	0	0

2 Comparison of the reaction force at 50 % compression of test specimen

	Reaction force at 50 % Compression	Difference to reference in percentage
Test	16355 N	reference
Optimisation Abaqus/Isight	16160 N	-1.19 %
Optimisation AnsysWB/OptiSlang	16678 N	+1.97 %

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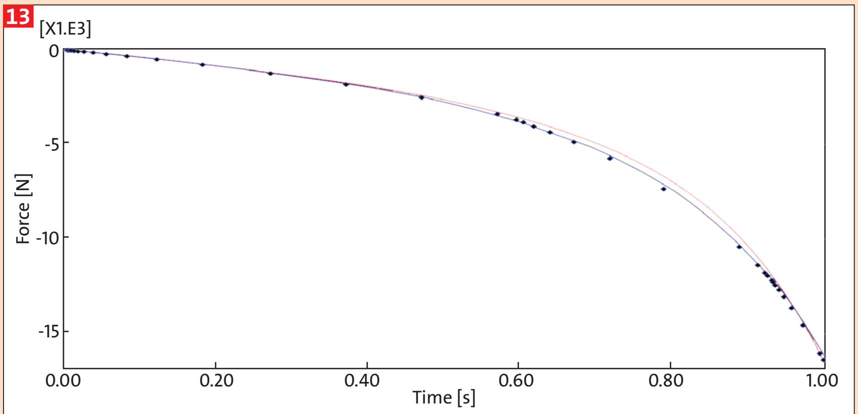

Fig. 13: Comparison of the reaction force; test data (black), Aabqus/Isight (blue), Ansys-WB/OptiSlang (red).

stopped after 58 min and 379 iterations by the abort criterion. The calculation was executed on a PC with the following performance characteristics:

- Intel Xeon @ 3.6 GHz (4 Cores)
- 64 GB Ram
- Windows 7

Optimisation results using alternative software

In addition the task was also solved by using Ansys Workbench and OptiSlang as alternative commercial software products to Abaqus and Isight. The starting situation remains the same, though. The result of both material parameter optimisations are depicted in Fig. 13.

Conclusion

The described reverse-engineering-process demonstrates how nonlinear optimisation of material parameters helps to obtain material data effectively and economically. The optimisation is fully computer based and allows the use of simple compression test setups including friction. Besides rather cheap computation time for the optimisation, there are other minor tasks to be done prior to the actual parameter optimisation. A rather simple FE model has to be created which represents the compression test geometry. Additionally, a simple compression test of a rubber specimen needs to be performed in order to get the initial data (reaction force over deformation) for the parameter optimisation. Since friction in between rubber specimen and clamping plate does not have to be avoided, the compression test can be kept simple, no tedious arrangements or expensive procedures are necessary as it would be in a low friction experimental setup. The optimised results match reality very favourably. Realistic simulations of critical industrial components made from elastomers can be provided by using this time and cost efficient computer based optimisation procedure for nonlinear material parameters. Eventually, it should be mentioned that the current procedure can also be applied to other classes of elastic materials besides elastomers.

References

[1] Ogden, R.W., Non-Linear Elastic Deformations. Dover Publications, Mineola, NY (1997).
[2] Yeoh, O.H., Some Forms of the Strain Energy Function for Rubber. Rubber Chemistry and Technology, Vol. **66** (5), pp. 754 (1993).
[3] Abaqus 6.14 Documentation, www.3ds.com/products-services/simulia/products/abaqus/.
[4] Hooke , R., & Jeeves, T.A., „Direct Search" Solution of Numerical and Statistical Problems*, Journal of the ACM (JACM), Vol. **8** (2), pp. 212, NY (1961).
[5] Deb, K. Optimization for Engineering Design. Algorithms and Examples New Dehli, Prentice-Hall (1995).