Tire · parameterisation · optimisation · finite element method · design of experiments

Tire models are used for several applications in the field of vehicle dynamics and tire construction. The parameterisation is a necessary step during the preparation of these models. This article describes the combination of the tire model RMOD-K FEM and the optimiser HOSPACK for executing a parameter optimisation, based on measured physical tire properties.

Parameter Optimization of Finite Element Tire Models

Introduction

Several applications in the field of vehicle dynamics and tire construction require the use of different tire models. For example, finite element tire models, with a high number of degrees of freedom (NDOF), are used by tire manufacturers to investigate tire performance and durability. On the other hand, OEMs require tire models linked to full vehicle models. These models, which are usually based on physical approaches, have a comparable small number of degrees of freedom in order to reduce the computational effort of time domain simulations.

Each model is based on a certain set of parameters to simulate the tire behaviour under different rolling conditions. Because of the several approaches these parameters are often incomparable, especially for the hybrid modelling approaches. The parameters of finite element models describe strictly physical values like material properties (stiffness, damping and density) and are comparable between different FE software systems, but these values are often not available. The reason therefore is that the tire manufacturers in many cases want to keep their internal know how — for instance in the case of race tires. That is the reason why the parameterisation of tire models is a necessary procedure for transferring the physical tire properties, measured or simulated, to the model side. This paper presents an approach for the parameter optimisation — a parameterisation combined with an optimiser — of a finite element tire model based on a few measured tire properties.

Design of experiments, an optimisation supporting tool

The design of experiments (DOE) is a well known method for the sensitivity analysis of model parameters and a feature of several commercial CAE programs. Besides the single sensitivity analysis it is possible to use the DOE as a step for preparing an optimisation run.

The main parts of an optimisation are the optimisation kernel (mathematical method) and the target function building element (optimisation subject). The optimiser interprets the target values and sets (new) values for the so called design variables. The target values itself indicates the quality of the set of design variables in the context of the target function. Most users have no detailed knowledge about the target function — mainly the shape is of interest to find proper initial values for the design variables, especially at the beginning of a completely new optimisation. Without the knowledge of the target function shape the user could not be sure that the target function is able to describe the optimisation problem correctly and which value denotes the best solution.

Figure 1 shows the shape of a function which depends on two variables, x and y. This function has two minima, a local one with the coordinates of x = -0.15 and y = 0.79 and the global one at x = -0.15 and y = -1.15. The left side of the figure shows the results of two DOE runs. Both runs are done with the same parameters, only the number of trail points is changed. The low order DOE (6x6 trail points) is not able to represent the right shape of the target function, only the global minimum is shown. The local one is not displayed by the rough trail point resolution. The higher order DOE (17x17 trail points) returns a detailed overview about the target function shape showing the local and the global minimum.

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Two optimisation results for the same function, OPT1 and OPT2, are shown on the right side of the figure. Both optimisation runs are executed with the same solver settings, only the initial values are changed. The trail points determined by the optimiser are the blue points and the initial values are marked by the green circle around the trail points. The initial value of the first run is located at the left corner of the target function and the optimisation returns the local minimum as the solution. The second run starts at the right corner and returns the global minimum as the optimum solution. Without the knowledge of the target function shape the user may accept the results of the first run, where the optimiser returns the solution of the local minimum.

That example shows that the usage of a DOE as a pre-processing step of an optimisation could improve the results quality and the user is able to rank the optimisation results. Another advantage is the possibility to get an impression of the target function itself. This is also valid if more than two design variables appear in the results of the first run, where the optimiser returns the solution of the local minimum.

Building the Target function

The target function denotes the quality of an optimised result compared to reference values. The reference values for the parameterisation of tire models are taken from measurements of the physical tire properties, for example the global vertical, lateral and tangential stiffness, the vibration modes, rolling resistance and contact path dimensions or contact patch pressure distributions and local tire deformations under load.

The computation of the target function in this paper is restricted to two physical tire properties, the vibration modes and the global vertical stiffness in order to keep the potential measurement effort small. The vibration modes describe the dynamic behaviour of the tire and the global vertical static stiffness defines the tire deformation under a vertical deflection.

The main idea for the definition of a target function is to compute the difference between the reference set of measurement results and a current set derived by model results. That definition is typical for a minimisation problem and the best possible target value of the target function is zero. The target function part of the vibration modes describes the relative difference between the measured and the simulated eigenfrequencies – see equation (1). The mode number is indicated by \( i \).

\[
\Delta f_i = \frac{f_i^{(mea)} - f_i^{(sim)}}{f_i^{(mea)}}
\]

The effect of the mode shifts has to be considered during the computation of the target function: usually the results of a modal analysis are sorted by the frequency in an ascending order. Changes of the stiffness and density distribution could lead to a different mode ordering. The sidewall stiffness for instance influences the rigid body modes while the belt stiffness influences the elastic modes. A changed stiffness distribution could switch the order of two subsequent modes. Therefore the mode shapes have to be checked and in some cases reordered before the straightforward computation of the frequency differences is carried out. To compare different mode shapes, the modal assurance criterion (MAC) \([1]\) offers a reliable measure. The MAC value describes the similarity of two eigenvectors. Similar eigenvectors generate MAC values near one, the MAC value of orthogonal eigenvectors is zero. By using this criterion the simulated modes will be ordered properly with respect to the measured modes and the frequency difference could be computed directly.

The global vertical static stiffness is measured by the vertical force as function of the tire vertical deflection. The simulation uses the same deflection values, in a finite element analysis normally the ground deflection. Then, the measurements and the computed vertical forces are directly used for the computation of the relative error between the measured and simulated value, see (2).
Equation (3) describes the computation of the multi target function value by taking into account dynamic and static results for a certain number of considered mode shapes \( N \) and vertical stiffness points \( M \). One important point is the square root in the formulation of the relative vertical force difference.

\[
\Delta F_i = \left[ \frac{F_{i}^{\text{max}} - F_{i}^{\text{min}}}{F_{i}^{\text{max}}} \right] \quad (2)
\]

\[
J_{\text{err}} = \frac{1}{2} \sum_{i=1}^{N} \Delta F_i + \frac{1}{M} \sum_{j=1}^{M} \sqrt{\Delta F_j} \quad (3)
\]

On one hand, the vertical force is directly proportional to the vertical stiffness. On the other hand, the relations between the eigenfrequency and the stiffness of a single mass-spring system is not proportional, see (4).

\[
\omega = \sqrt{\frac{c}{m}} \quad (4)
\]

To use both properties with similar weights the square root is added to the vertical force parts. Without that correction the vertical force errors have greater influence on the target function than the eigenfrequencies. When other target objects are added to an expanded multi target function, the functional weights in the sense of \( \Delta \) and \( k \in \mathbb{R} \) must be chosen carefully.

**RMOD-K FEM**

RMOD-K FEM is a single purpose finite element code to support research and development in the field of tire mechanics, which was developed at the University of Applied Sciences in Brandenburg. The whole system is separated into the graphical user interface (GUI) and two mathematical solvers. The first solver carries out the calculations in the time domain, like nonlinear dynamic and nonlinear static analysis. Additional results, like the steady state and the modal analysis could also be computed by using that solver [2], [3]. The second solver allows computations in the frequency domain, like transfer functions.

The GUI is designed to support the pre- and post-processing of finite element tire models enabling the user to generate a finite element tire model intuitively with a comparable small time effort. Besides usual results like nodal properties as function of load or time, the post-processor offers additionally and easy to access result plotting options. Some of them are Gaussian point chains in circumferential direction at a certain lateral position (displacement, velocity, acceleration, stress, strain), 3D plots with a number of lateral positions and contact patch information like tangential and normal stress as well as a sort of grip level. One additional feature of the GUI is the possibility to define design variables and the usage of these variables for most of the pre-processor or analysis properties. The execution of a DOE or an optimisation is based on that feature.

A DOE could be directly created by the RMOD-K FEM - GUI. Therefore, the user has to carry out the following steps:

- define the design variables and their ranges
- use the design variables as model properties
- build and run the DOE

A DOE usually uses a large number of evaluations, depending on the number of design variables and the number of design steps. To reduce the simulation effort the DOE tool supports the parallel evaluation on multi-core machines. Each core executes a single finite element run, the simulation itself is not necessary parallelised. After the execution of a DOE a usual step is the post-processing with an additional tool. The DOE-evaluator is able to store all DOE results, computes all target functions and visualises the target function shape. The tool uses the same algorithm for computing the target function like the optimisation application. The GUI allows also the preparation and the execution of the optimisation. The necessary steps are nearly the same compared to the DOE.

**HOPSPACK**

The optimisation tool itself is an independent application which is decoupled from the GUI. The optimiser used in the following examples is the Hybrid Optimisation Parallel Search PACKage (HOP-
The main reason for choosing that optimisation tool is the availability of the C++ source code, which allows a direct link between the optimisation environment of HOPSPACK and the computation of the target function, including RMOD-K FEM as a dynamic link library. The second reason is the derivative free optimisation approach: the computation of the target function has not to be extended to the computation of its first derivative. An additional feature is the possibility of a parallel execution of HOPSPACK. That allows the parallel execution of several finite element runs, which decreases the complete simulation time of the full optimisation run highly compared to a single core execution. The parallel execution requires the message passing interface (MPI) [5].

Figure 2 shows the structure of the optimisation environment. The Citizens are the optimisers itself, the Conveyor executes and stores all target function values and the Mediator is the interface between the both main parts. The computation of the target function and the RMOD-K FEM solver are directly integrated to the Conveyor. Based on that open structure, it is also possible to add a custom optimisation algorithm to the whole project in the same way like the HOPSPACK ones, the GSS and the GSS-NLC optimisers.

The numerical properties of the HOPSPACK optimisation algorithm could be modified in order to improve the optimisation results and shorten the computational time. The related parameters, their defaults and meanings are described in detail in the HOPSPACK manual.

Based on the defaults the influence of the parameters is investigated for a parameter optimisation of a tire model. For a first test case the physical tire properties are calculated with the RMOD-K FEM model (instead of measurements) and the initial design values are changed. Because of the usage of computed properties, there is a design set which generates a target value near to zero. These investigations show that there is the possibility to improve the optimisation results in terms of accuracy for example by using a smaller step tolerance or a smaller contraction factor. On the other hand, the simulation time than increases. Changing the numerical values lead always to a conflict between the results quality and the simulation time. The HOPSPACK defaults generate acceptable computational effort with respect to the results quality and are used for all following optimisations. However, for improving the results it is also possible to use a cascade optimisation. The solution of the first optimisation run is used as the initial state for the second run and the design variable bounds are also adjusted (normally decreased). Decreasing design variable bounds has the same influence on the optimisation like changing the numerical parameter, but the user is able to influence the optimisation between the single runs.

Uniqueness of the target function – test case
Besides the optimiser and the corresponding settings, the target function properties have the great influence on the results and on the process of an optimisation run. Figure 3 shows the target function shape of a DOE, where the matrix and the carcass Young’s modulus are varied. The computation of the target function – shown in figure 3 – considers only the results of the global vertical stiffness, not the eigenfrequencies.

The target function is not smooth and not unique. While at higher values of the matrix Young’s modulus, the direction towards the minimum seems unique, at lower values of this property, there are more than two local minima. That means that the same vertical static stiffness could be reproduced by the tire model with more than one set of design variables. For example a data set with a low matrix and a high carcass Young’s modulus generates nearly the same vertical stiffness like a data set with high matrix and a low carcass Young’s modulus. Consequently, the optimisation results may depend a lot on the initial design values and the results are not unique. This is a strong hit towards an
incomplete target definition which may be hidden if the target function shape is unknown.

This example shows that a single domain target function is not in any case unique. Different design sets lead to a similar behaviour. Adding orthogonal experiments lead to a proper target function definition. Consequently, the modified target function – shown in figure 4 – also considers the eigenfrequencies. This shape is much more unique compared to the first shown shape and there is only one local minimum in the area of investigation.

Results

The parameter optimisation is done for two tires and related models. The first finite element tire model (tire A) uses some assumptions, especially for simplifying the tire model. The whole tire model uses only one Young’s modulus of the rubber matrix and the local distribution of the rubber properties is neglected. The design variable bounds are not taken from material testing information.

The belt angle for both tires is given and the layer thickness is fixed. The parameter optimisation uses three design variables: the matrix (rubber), the carcass layer and the belt layer Young’s modulus. The target function considers the vertical stiffness and the unloaded or loaded state eigenfrequencies at one single inflation value. The optimisations are executed on a MPI-cluster with 16 cores. The tire A has the dimension of 235/45-R20, the mesh consists of 7020 elements (HEX8) with 27000 NDOF. The comparable small mesh dimension is used with respect to the results begin global tire properties with a very small result sensitivity on the mesh size. If local properties like stress or strain are taken into account, a much more detailed mesh is required. The parameter optimisation takes the vertical stiffness and the unloaded modal system at an inflation of 2.5 [bar] into account.

The first trial returned optimisation results with a minimum target function of 15 %. The results for the three design variables are:

- matrix Young’s modulus: 5.35 N/mm²
- carcass Young’s modulus: 8000 N/mm²
- belt Young’s modulus: 10 000 N/mm²

The simulation used 443 trail points with a calculation time of 206 minutes. The results for the vertical forces are really accurate, the average difference is about 2 %, see figure 7 on the left side. Figure 5 shows the difference in case of the eigenfrequencies, the average aberrations about 25 % are not acceptable. The tire model is only able to represent the vertical stiffness accurately, not the eigenfrequencies. Because of that result the DOE tool is used to investigate the target function. The assumption of one matrix Young’s modulus prevents a better solution for both, the vertical stiffness and the eigenfrequencies for that modelling. The next trial is done with the cross section divided into three areas with different material properties: the belt, the upper sidewall and the lower sidewall matrix Young’s modulus. Consequently, the optimisation uses five design variables and the resulting target function value could be reduced from 15 % to 7.4 %. The average error of the vertical force is still the same and the difference of the eigenfrequencies could also be reduced from 25 % to 13 %.

For a further improvement of the modal properties the mass distribution could also be considered as design variable.

The tire second tire – tire B – is of the dimension of 225/50-R16, the mesh consists of 3480 elements (HEX8) with 15480 NDOF. The parameter optimisation used the vertical stiffness and the loaded modal system at an inflation of 2.6 [bar]. The assumption of only one matrix Young’s modulus is still active.

The optimisation ends with target value of 2 % and the results for the three design variables are:

- matrix Young’s modulus: 5.35 N/mm²
- carcass Young’s modulus: 8000 N/mm²
- belt Young’s modulus: 10 000 N/mm²

The simulation used 443 trail points with a calculation time of 206 minutes. The results for the vertical forces are really accurate, the average difference is about 2 %, see figure 7 on the left side.
- matrix Young’s modulus: 22.3 N/mm²
- carcass Young’s modulus: 356 N/mm²
- belt Young’s modulus: 5088 N/mm²

The simulation needed 70 minutes for the calculation of 744 trail points. Because of the smaller model size the simulation time is much lower compared to the first tire model. The average difference between the measured and computed vertical forces and eigenfrequencies is about 2%. Figure 6 shows the difference of the eigenfrequencies and the vertical forces are shown in figure 7 on the right side. It is also possible to improve the optimisation results by suppressing the simplifying assumptions, like done with the tire A, but the current results are still accurate enough.

Conclusions

Building a target function, some decisions must be taken concerning the number of inflations in the case of vertical stiffness and the number and type of modes. In figure 7 the computed vertical stiffness for both models at different inflations compared with the measurements are shown. The optimisation run itself considers only the vertical forces of the second inflation at 2.5 respectively 2.6 [bar]. The optimized tire models offer an acceptable accuracy for the vertical forces also at the other two inflations: to reduce the computational effort of an optimisation it is sufficient to use the vertical stiffness behaviour at one single inflation.

For improving the target function uniqueness as already mentioned, modal system data has also to be used for the optimisation. The number of modes to be included into the target function as well as the type of modes (rigid body or elastic) is also a variable in the definition of the target function, which has a potential influence on the uniqueness and convergence. The sidewall stiffness has a great influence on the rigid body modes while the belt stiffness influences the elastic modes a lot. Because of that behaviour a reduction of the considered modes leads to a stiffness transfer between the matrix and belt Young’s modulus, see figure 8. A set of five reference modes includes the rigid body and elastic modes while the single reference mode only takes into account the first rigid body mode. The decreasing mode number leads to a decreasing belt Young’s modulus and an increasing matrix modulus — a stiffness transfer effect. To get reasonable optimisation results, especially concerning the material properties, the eigenfrequencies of the rigid body and the elastic modes have to be considered.

Summary and further development

A useful tool for the preparation of a parameter optimisation is the DOE combined with an evaluation of the target function. It allows the user to set useful design variable initial values and bounds and to check the shape of the target function. It is possible to integrate the prior derived DOE results into the optimisation in order to decrease the computational effort and the simulation time. The necessary reference measurements for the parameter optimisation are the vertical stiffness and the resonance frequencies. Additionally, measurements could improve the target uniqueness and the convergence behaviour. Any approach based either only on static measurements or only on dynamic experiments is incomplete in terms of uniqueness if no other sources of information are supplied - for instance “hard” design data.

The main focus of the further development lies on the investigation of the uniqueness of the target function. Therefore, it is planned to extend the computation of the target function for considering additional available tire data like the footprint area, the footprint pressure and lateral as well as tangential static tire stiffness. The influence of each measurement on the target uniqueness will be used for developing a guideline, which describes the necessary measurements for a full parameter optimisation. An additional topic is the integration of the DOE results to the optimisation run for reducing the computational effort and the full simulation time. At the end there will be a complete procedure for the parameter optimisation of a finite element tire model, which includes all the preparing steps of the tire model, a guideline of the required measurements and a post-processing step, which delivers the parameterised tire model.

Literature