Efficient Material Parameter Calibration of Elastomer Specimen in uniaxial Tension, planar Shear and equibiaxial Tension

Introduction
The retrieval of material parameters for nonlinear materials such as elastomers and thermoplasts is a key aspect in the optimization of critical industrial components and thus, their cost. By use of these parameters in finite element simulations, geometries can be analyzed and subsequently adjusted to yield optimal performance.

For describing arbitrary quasi-static 3D stress states in solid rubber compounds, it is necessary to conduct three different experiments. By means of uniaxial tension, equibiaxial tension and planar shear tests, the necessary data can be acquired and employed to retrieve above mentioned material parameters.

However, material parameter calibration based on uniaxial tension, equibiaxial tension and planar shear tests assumes homogeneous stress states in the test specimens during loading. For uniaxial tension and planar shear tests homogeneous stress distributions can be easily achieved, whereas in equibiaxial tension tests a purely homogeneous stress state in the test specimen is hard to realize.

The current paper proposes a method to cure stress inhomogeneities in equibiaxial tension tests by a numerical optimization procedure after testing. Thus, simple test setups can be realized which allow for efficient material data acquisition. In a previous publication by Eberlein [1], a similar method was proposed for friction biased uniaxial compression tests that allow a proper replication of compression dominated stress states in critical industrial components. This paper shows a generalization of such an approach for an accurate description of arbitrary quasi-static 3D stress states in solid rubber compounds.

In the current investigation three different solid rubber compounds (EPDM) are tested and calibrated including a material parameter optimisation. The experiments are conducted at room temperature (20°C) and for one material additionally at 60°C.

Test setup
The tests are run on a biaxial test machine that additionally includes a thermal chamber. With this chamber, temperatures from -50°C up to 250°C can be set. Strain measurements are obtained by using a video extensometer, i.e. a camera which measures the distance...
between two markers on a specimen over time (see Figure 1). During post processing corresponding engineering strains are calculated. The force measurements are done via load cells. The geometry of the samples is measured using a micrometer. The corresponding engineering stress is subsequently calculated from force and cross sectional area data. All samples are fixed in the machine by clamping plates, which are tightened with a specific torque that is high enough as to avoid slipping of the samples while testing.

**Uniaxial tension**

For uniaxial tension tests as shown in Figure 1, samples are punched from raw material plates according to DIN EN ISO 527-2 Type 1BA. This type of specimen is usually used for thermoplastic material testing but was employed in the current case for solid EPDM rubber compounds. The thickness and width are measured at three points, whilst the width is measured once. The samples are then clamped on the wide side and pulled in vertical direction which results in a plane strain state in the middle of the sample (compare with Bergström [2]), since the width is much larger than the height. In case of the current test setup, a ratio of width to free sample height of 10 is used. As an example the previously discussed deformation states are shown in Figure 3 for \( \alpha = 0.1 \).

**Planar shear**

For the planar shear tests as shown in Figure 2, samples with a height of 40 mm and a width of 100 mm are cut from raw material. The thickness of the samples is again measured at three points, whilst the width is measured once. The samples are then clamped on the wide side and pulled in vertical direction which results in a plane strain state in the middle of the sample (compare with Bergström [2]), since the width is much larger than the height. In case of the current test setup, a ratio of width to free sample height of 10 is used. As shown by Bergström [2] the plane strain state can be transformed into a pure shear state. Therefore the test is called planar shear test. In order to prove this, the deformation gradient for incompressible plane strain tension we find:

\[
F_{\text{plane}} = \begin{pmatrix} 1 + \alpha & 0 & 0 \\ 0 & 1 + \alpha & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

wherein we have the same \( \alpha \) as above. We have no stretch in the 3-direction.

As we rotate this gradient by 45° around the 3-direction, we end up with:

\[
F_{\text{plane, rot}} = \begin{pmatrix} 1 + \alpha^2 & \alpha & \alpha^2 + O(\alpha^3) \\ \alpha & 1 & \alpha^2 + O(\alpha^3) \\ \alpha^2 + O(\alpha^3) & \alpha & 1 \end{pmatrix}.
\]

\( O(\alpha^3) \) is indicating the remainder of the term to be of order 3. For \( \alpha \ll 1 \) we can ignore second order term and higher in \( \alpha \), leading to:

\[
F_{\text{plane, rot}} \approx \begin{pmatrix} 1 & \alpha & 0 \\ \alpha & 1 & 0 \\ 0 & 0 & 1 + \epsilon \end{pmatrix} = F_{\text{shear}}.
\]

The value \( \epsilon \) is a small value needed to enforce incompressibility. Therefore \( F_{\text{plane, rot}} \) is equivalent to the deformation gradient for pure shear as described above. As a consequence pure shear and plane strain states are the same for small strains by assuming incompressible material behaviour. As an example the previously discussed deformation states are shown in Figure 3 for \( \alpha = 0.1 \).

**Equibiaxial tension**

Equibiaxial tension tests (see Figure 4) are conducted with square samples with a side length of 100 mm, also cut from raw material. In Bergström [2], it is shown that equibiaxial tension tests are equiva-
lent to uniaxial compression tests as long as the material considered can be approximated as incompressible. This basically holds for the tested EPDM rubber compounds. All raw material plates considered had the same thickness of 2 mm. However, for equibiaxial testing, significant stress inhomogeneities occur within the samples during loading (see Figure 5). These stress inhomogeneities increase the sample stiffness compared with purely homogeneous stress states. A proper material calibration must be based on purely homogeneous stress states, though. Indeed, it proves to be quite challenging to realize an undisturbed homogeneous stress distribution in an equibiaxial test sample. On the other hand it is much more efficient to correct the inhomogeneous stresses occurring in an equibiaxial tension test setup as shown in Figure 4 by a subsequent numerical optimisation procedure as shown below.

**Test parameters**

All experiments are conducted under strain control. For each test setup, a specific strain goal (see Table 1) is defined and the machine is set up to pull up to this ratio, referring to the initial distance between the clamps.

The experiments are conducted with a speed of 10 mm/min (5 mm/min per clamp), which is considered quasi-static. Since the material behaviour changes after being loaded for the first few times (virgin samples vs. preconditioned samples), the samples are stretched three times to the strain goal before being used for final testing. This procedure is conducted at room temperature for all samples and those tested at 60°C. Samples rested for an hour at test temperature before the actual test to ensure a homogeneous temperature distribution.

To avoid sample slipping effects during loading, all samples are clamped with a specific torque prior to testing. For each material, test setup and temperature, three samples are tested. In preparation of the material calibration process, the average response of three samples (engineering strain and engineering stress) is calculated. Additionally, force displacement data of the biaxial tension test is also averaged for the following optimisation step.

To ease the material calibration process, all data from the experiments (which are recorded at 50 Hz) are reduced to 200 data points per average curve, or even to 50 for the averaged force displacement data.

**Hyperelastic material models**

For accurately describing finite elastic strains (hyperelasticity) in solid rubber compounds the well established Ogden model is applied for material parameter calibration and optimisation throughout this paper. Here, it is employed in specific
form as a third order strain energy potential (compare with Abaqus [3]):

\[ U = \sum_{i=1}^{N-2} \frac{2\gamma_i}{\alpha_i} \left( \lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3 \right) + \sum_{i=1}^{N-3} \frac{1}{D_1} (I_{str-1})^{\alpha_i}, \]

where \( \lambda_i = J^{-1/3} j_i \), \( j_i \) are the principal stretches and \( J \) is the volume ratio:

\[ f \equiv \det(F) = \det \left( \frac{\partial \mathbf{x}}{\partial \mathbf{X}} \right). \]

\( F \) is the deformation gradient tensor.

For the compressibility we suppose almost incompressible materials with \( K_0 \) as initial bulk modulus:

\[ D_1 \equiv 0.02 \frac{1}{MPa} \Leftrightarrow K_0 = 2 \frac{D_1}{D_1} = 100 \text{ MPa}. \]

During the optimisation process, the \( \mu_i \) and \( \alpha_i \) are varied. From (1), by assuming purely incompressible material behaviour, the homogeneous stresses for uniaxial tension, planar shear and equibiaxial tension can be obtained (compare with Bergström [2]):

\[ \sigma_{\text{uniax, true}} = \sum_{k=1}^{N} 2 \frac{\mu_k}{\alpha_k} (\lambda^{\alpha_k} - \left( \frac{1}{\sqrt{3}} \right)^{\alpha_k}), \]

\[ \sigma_{\text{planar, true}} = \sum_{k=1}^{N} 2 \frac{\mu_k}{\alpha_k} (\lambda^{\alpha_k} - \left( \frac{1}{\sqrt{3}} \right)^{\alpha_k}), \]

\[ \sigma_{\text{biax, true}} = \sum_{k=1}^{N} 2 \frac{\mu_k}{\alpha_k} (\lambda^{\alpha_k} - \left( \frac{1}{\sqrt{2}} \right)^{\alpha_k}). \]

For comparison, we also consider the Mooney-Rivlin model, wherein the strain energy potential is defined by:

\[ U = \sum_{i=1}^{N} C_0 (I_1 - 3) (I_3 - 3) + \sum_{i=1}^{N} C_1 (I_{str} - 1)^2. \]

The first and second strain invariant defined by:

\[ I_1 = \text{tr}(\mathbf{B}), I_2 = \frac{1}{2} [I_1^2 - \text{tr}(\mathbf{B} \cdot \mathbf{B})], \]

\( \mathbf{B} \) is the deviatoric part of the left Cauchy-Green strain tensor. Note that \( J_{el} = J \), if thermal expansion of the rubber material is neglected.

**Material calibration and optimisation process**

For the material calibration process, a material model has to be chosen. In a first step the Mooney-Rivlin model is selected. In addition, the Ogden model with rank three is also chosen to allow for an improved fit. After obtaining the experimental stress-strain-data, the calibration process is started using MCALIBRATION [4]. It fits all three test setups (load cases) at the same time, since it allows allocating different test setups (uniaxial, equibiaxial, etc.) to different data curves. For the material calibration process MCALIBRATION offers various numerical optimisation algorithms that are not described in detail here.

The difference between the material calibrations based on Mooney-Rivlin (Figure 6) and Ogden (Figure 7) is clearly visible in terms of fitting accuracy. For the Mooney-Rivlin fit, a significantly lower R2 determination coefficient is obtained compared with the Ogden fit.

After obtaining these first (calibrated) material parameters, an optimisation process (Figure 8) is started using FE [3] analysis. A finite element model of the equibiaxial test setup is designed that represents the stress inhomogeneities discussed above (also compare with Figure 5).

The material thickness is modelled according to the measured data from the experiments. The material parameters obtained by the material calibration process are used in the FE software to define the hyperelastic rubber material according to the Ogden model.

The optimisation process is based on force-displacement data. The model is controlled by the displacement of the four clamps, and the optimisation essentially compares the reaction forces obtained by the FE-model to those obtained experimentally. Initially, due to stress inhomogeneities, the reaction forces obtained by FE simulations based on previously calibrated material data do not coincide with experimental findings.

The FE simulations can be included in MCALIBRATION as a special load case. While running the optimization procedure, the experimental data from the **Flowchart of optimisation process.**

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**References:**

[1] Bergström, [2]...
planar shear and uniaxial tension tests are calibrated in parallel. Figure 9 depicts the initially calibrated curves already shown in Figure 7 and those obtained through the optimisation process. The uniaxial fits show the least discrepancy, which is unsurprising. The uniaxial test is the one nearest to the real, experimental stress state. For the biaxial curve, we find a rather big discrepancy from the calibrated data, caused by the FE optimisation process.

As mentioned before the calibration process is always based on the assumption of ideal, homogeneous stress states during the tests. Since this is not the case for all experimental tests (i.e. the equibiaxial case), the optimised curves for equibiaxial tension significantly deviate from the calibrated ones. Indeed, the material response in equibiaxial tension becomes much softer. However, the big advantage of optimised equibiaxial tension curves is that they represent purely homogeneous stress states in equibiaxial testing samples. Therefore the optimised material parameters can be used for accurately simulating arbitrary quasi-static stress states, whereas the calibrated data would provide too stiff results in compression dominated loading conditions of critical industrial components made from solid rubber compounds.

To validate the optimised curves, formulas (2) from above can be considered. They show the principal stress (i.e. homogeneous stress) vs. principal strain response of the Ogden model for the three test setups. If optimised material parameters are used in formula (2), the homogeneous equibiaxial stress response perfectly correlates with the optimised equibiaxial tension curve (see Figure 10). This proves that the optimised equibiaxial tension curve indeed represents a homogeneous biaxial stress state.

Eventually, it should be mentioned that the FE model based optimisation process as described above can efficiently be run on a standard desktop computer. If material parameters of several compounds need to be optimised at the same time, this can most conveniently be achieved in subsequent or parallel optimisation runs over night.

Conclusion
In this paper an efficient material calibration method is presented that allows the retrieval of accurate hyperelastic material parameters accounting for solid elastomer compounds. The method uses simple experimental test setups for uniaxial tension, equibiaxial tension and planar shear. In combination with a FE based optimisation process stress-strain data based on homogeneous stress states can be derived. Thus, accurate modelling of arbitrary quasi-static stress states in critical industrial components made from solid rubber compounds is guaranteed. In addition, the presented method is very time and cost efficient, since the experiments can be prepared in a simple way and the optimisation process can be handled in a fully automated way on a standard laptop or desktop computer.

References: